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Multifractual Analysis of the Southwestern Iceland Sea Surface Mixed Layer Thermal Structure

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1 Introduction

We use a thermistor chain data set from the Royal Navy's Admiralty Research Establishment (ARE) to investigate the multifractal characteristics of the mixed layer thermal structure in the southwestern Iceland Sea near 69°N, 18°W in summer 1987 (Fig. 1). The track was a single 200 km straight-line tow of a 380 m long thermistor chain near the eastern edge of the east Greenland Current [*Scott and Killworth*, 1991]. The data show the existence of chimneys, indicating the nonstationary and intermittent processes in the ocean mixed layer (OML). Here, we will investigate the characteristics of intermittency and nonstationary in this thermistor chain data using the multifractal analysis.

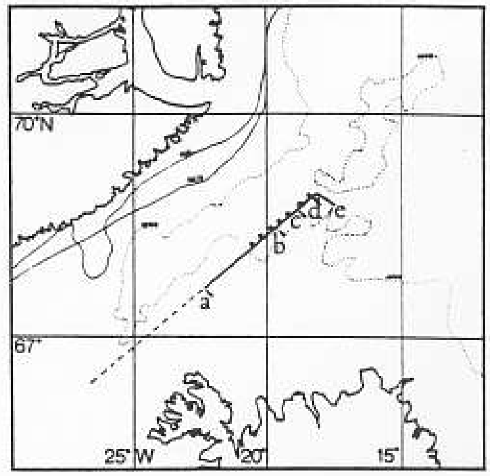

Figure 1 – The track along which the thermistor chain data were taken (after Scott and Killworth, 1991).

2 Thermistor Chain Data Set

A thermistor chain with 100 sensor pods along a 380 m straight-line in vertical was towed at a speed of 4 knots, sampled every 0.9 s, and gave a temperature profile about every 2 m horizontally [*Scott and Killworth*, 1991]. Each sensor pod along the chain measured temperature, and about one in five also measured pressure. This allowed us to deduce the depth distribution of the temperature data. Figure 2 shows contour plot of temperature ranging from -2°C to 8°C with 0.2°C increment on a vertical cross-section between the two marked locations 'b' and 'e' (Figure 1). As pointed by *Scott and Killworth* [1991], the data set shows a highly irregular nature, which is a characteristic of the Denmark Strait region.

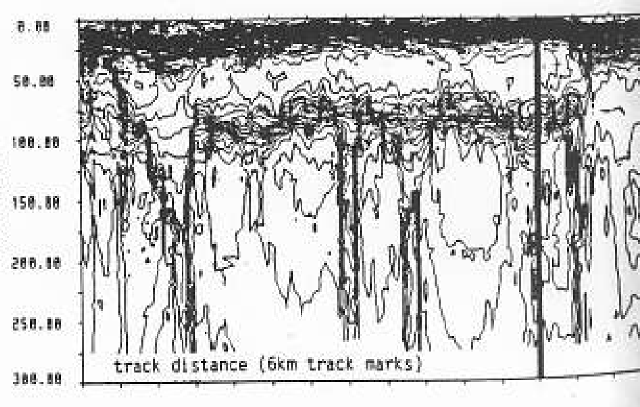

Figure 2 – Thermistor chain data from the section of track marked b-e in Figure 1 (after Scott and Killworth, 1991).

3 Data Stationarity

What is the inherent variability of this high resolution temperature data? What is the statistical properties? First, we should investigate the data stationarity. For a given depth, the temperature data is a function of the horizontal coordinate, x,

$$T_i = T(x_i), \quad x_i = il, \quad i = 0, 1, ..., \Lambda, \quad \Lambda = L/l \quad (1)$$

where $l = 2$ m, is the horizontal resolution of the data, and L is the total horizontal scale of the data set. Our data set has 24,000 temperature profiles, that is, $\Lambda = 24,000$.

The data stationarity is usually tested by 'power law', that is, to compute the power spectrum of the data $T(x_i)$,

$$E_j = E(k_j), \quad k_j = j/L, \quad j = 1, 2, ..., \Lambda/2. \quad (2)$$

For a scaling process, one expects power law behavior (*Davis, et al.*, 1994],

$$E(k) \propto k^{-\beta} \quad (3)$$

over a large range of wavenumber k. *Mandelbrot* [1982] and *Schertzer and Lovejoy* [1987] argued that the spectral exponent β contains information about the degree of stationarity of the data. *Davis et al.* [1994] found that if $\beta < 1$, the field is stationary; if $1 < \beta < 3$, the field contains nonstationary signal with stationary increments and in particular, the small-scale gradient field is stationary; if $\beta > 3$, the field is nonstationary.

We computed the power spectrum of the temperature data for each depth. The spectral exponent is

$$1 < \beta < 2 \quad (4)$$

which means the data field is nonstationary with stationary increments. Figure 3 shows an example of the power spectrum for 40 m depth. Thus, the temperature field is nonstationary process with stationary increments. We may use the structure function and singular measure to investigate the features of such field [*Davis, et al.*, 1994].

4 Structure Functions

Since the thermistor chain data set has stationary increments, we should study the statistical characteristics of the gradient field,

$$|\Delta T(r; x)| = |T(x_{i+r}) - T(x_i)|, \quad i = 0, 1, ..., \Lambda - r. \quad (5)$$

The qth-order structure function is defined by the mean of the qth power of $|\Delta T(r; x)|$,

$$S(r, q) \equiv \langle |\Delta T(r; x)|^q \rangle = \frac{1}{\Lambda - r} \sum_{i=0}^{\Lambda - r} |\Delta T(r; x)|^q. \quad (6)$$

Here $r \sim 1/k$, is the lag between two data points. For example, $r = 1, q = 1$, the structure function represents the average gradient, i.e.,

$$S(1, 1) = \frac{1}{\Lambda - 1} \sum_{i=0}^{\Lambda - 1} |T(x_{i+1}) - T(x_i)|. \quad (7)$$

In order to show the dependence of the structure function $S(r, q)$ on r and q for the thermistor chain data, we plot $Log_2[S(r, q)]$ (vertical axis) versus $Log_2(r)$ (horizontal axis) for different q-values from 0.5 to 4.0, as shown in Figure 4. We may use straight lines with different slopes for each q, which means that the structure function for the mixed layer temperature in the southwestern Iceland Sea satisfies the power law

$$S(r, q) \sim r^{\zeta(q)} \quad (8)$$

with the exponent $\zeta(q)$ depending on q. Our computation (Figure 5) agrees quite well with earlier studies [*Frisch*, 1991; *Marshak et al.*, 1994]: $\zeta(q)$ is monotonically and near-linearly increasing with q. Thus, we may represent $\zeta(q)$ by

$$\zeta(q) = H(q)q \quad (9)$$

where $H(q)$ is nearly a constant.

For $q = 1$, the structure function does not depend on r if $H(1) = 0$, which leads to the exact stationary. Besides, as long as $\zeta(1) = H(1) > 0$, the temperature field $T(x)$ is stochastically continuous, that is, when r is small, $|T(x_{i+r}) - T(x_i)|$ is too. Furthermore, *Mandelbrot* [1977] showed that

$$H_1 = H(1) = \zeta(1) = 2 - D_{g(T)} \geq 0 \quad (10)$$

where $g(T)$ is the graph of $T(x)$ and $D_{g(T)}$ is the dimension of the graph $g(T)$. For an everywhere differentiable function $T(x)$,the graph $g(T)$ does not have fractal dimensions,

$$D_{g(T)} = 1$$

which shows a smooth curve with the dimension of 1. If the graphs $g(T)$ fill the whole space (exact stationary),

$$D_{g(T)} = 2$$

which leads to $H_1 = 0$.

5 Singular Measure

To investigate the intermittency of the data, we computed the small scale differences

$$\Delta T(1; x_i) = T(x_{i+1}) - T(x_i), \quad i = 0, 1, ..., \Lambda - 1, \quad (11)$$

its normalized values

$$\varepsilon(1; x_i) = \frac{|\Delta T(1; x_i)|}{\langle |\Delta T(1; x_i)| \rangle}, \quad \langle |\Delta T(1; x_i)| \rangle = \frac{1}{\Lambda} \sum_{i=0}^{\Lambda - 1} |\Delta T(1; x_i)| \quad (12)$$

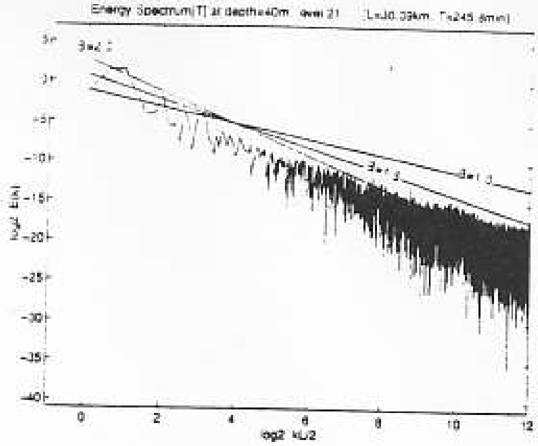

Figure 3 – **Power spectrum of the thermistor data at 40 m depth.**

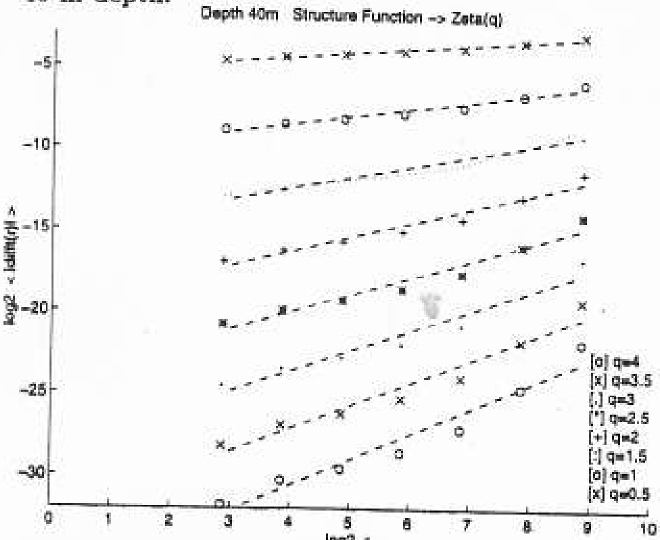

Figure 4 – **Structure functions for different q values.**

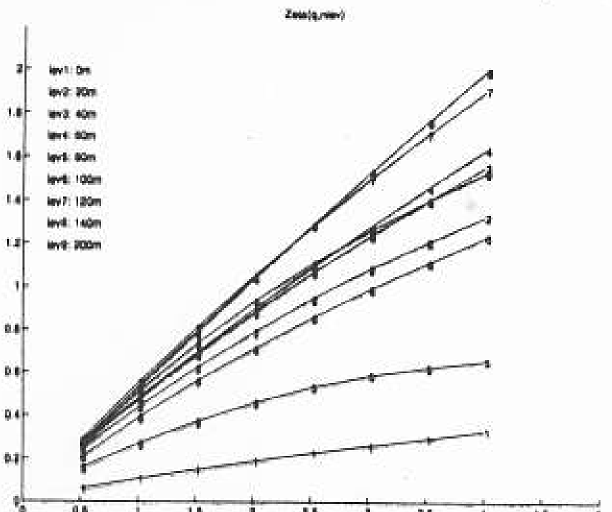

Figure 5 – **The zeta functions for different depths.**

and the averaged values

$$\varepsilon(r; x_i) = \frac{1}{r} \sum_{j=i}^{i+r-1} \varepsilon(1; x_j), \quad i = 0, 1, ..., \Lambda - r \qquad (13)$$

which is called the singular measure. *Davis et al.* [1994] shows that $\langle \varepsilon(r; x_i)^q \rangle$ has a power law dependence on scale r for many natural processes. To see if their viewpoints are valid for our thermistor chain data, we plot $Log_2 \langle \varepsilon(r; x_i)^q \rangle$ versus $Log_2(r)$ for different q-values at 40 m depth (Figure 6). The relationship between the two is represented by straight lines with different slopes, which confirm the power law

$$\langle \varepsilon(r; x_i)^q \rangle \sim r^{-K(q)}, \quad q \geq 0. \qquad (14)$$

The dependence of the slope on q (Figure 6) indicates that the exponent $K(q)$ is a function of q.

We have plotted $K(q)$ versus q for the thermistor data at different depths (Figure 7), and notice that

$$K(0) = K(1) = 0 \qquad (15)$$

and the convexity

$$\frac{d^2 K(q)}{dq^2} > 0 \qquad (16)$$

for all q. These features follow directly from the definition [*Davis et al.*, 1994]. We also have

$$K(q) < 0 \quad \text{only if} \quad 0 < q < 1 \qquad (17)$$

which reflects the fact that, in this range, taking a qth power necessarily reduces the fluctuation of $\varepsilon(r; x_i)$; and otherwise

$$K(q) \geq 0, \quad \text{if} \quad q \geq 0 \qquad (18)$$

Following *Grassberger* [1983] and *Hentschel and Procaccia* [1983] we may define a function

$$C(q) = \frac{K(q)}{q-1}. \qquad (19)$$

For $q \to 1$, we can use *l'Hosptal's* rule to define a straightforward measure of inhomogeneity in the sense of singular measure [*Davis et al.*, 1994]:

$$C_1 \equiv C(1) = K'(1) \geq 0 \qquad (20)$$

which is called the intermittency parameter. The larger the value of C_1, the more the intermittency and singularity the data set has.

6 Multifractal Plane

The parameter C_1 measures directly the degree of intermittency in the system, while H_1 directly measures its degree of nonstationarity. The plot of C_1 versus H_1, called the multifractal plane, shows the degree of nonstationarity and intermittency. Figure 8 shows the (H_1, C_1) values for several different depths of the thermistor chain data. We find much larger depth variation in H_1 than in C_1. All the values of C_1 are quite close to 0.1. However, the values of H_1 have a large variation. Besides, the nonstationarity decreases from the surface mixed layer water (20 m, 40 m, and 60 m) to the thermocline water (80 m, 100 m).

7 Conclusions

The multifractal analysis provides a useful framework for analyzing ocean data when complex nonlinear processes are at work. The upper layer thermal structure in the southwestern Iceland Sea has the following features:

(1) The spectral exponent is between 1 and 2, which means the data field is nonstationary with stationary increments.

(2) The inttermitency and singularity parameter C_1 is around 0.1 with a very small variation in depth.

(3) The stationarity parameter H_1 has a large variation, changing from high values (0.45-0.5) in the mixed layer to low values (0.28-0.4) in the thermocline.

8 Acknowledgments

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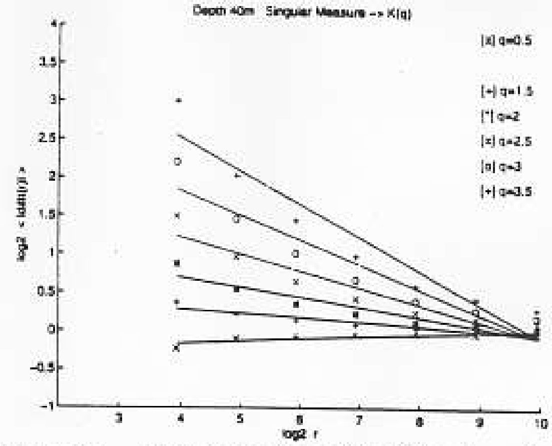

Figure 6 – Obtaining the K(q) function for 40 m depth data.

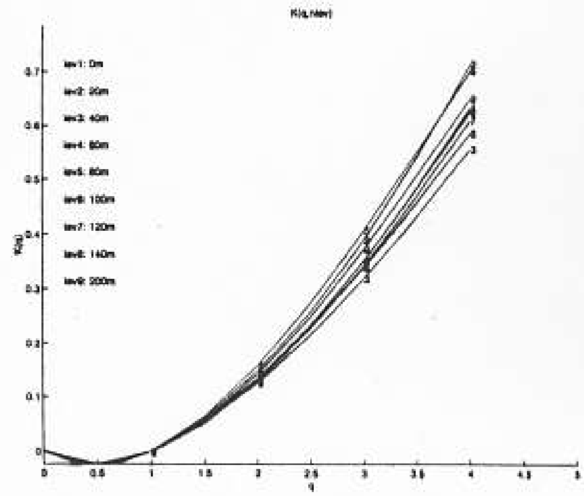

Figure 7 – The plot of K(q) against q from the surface to 200 m depth.

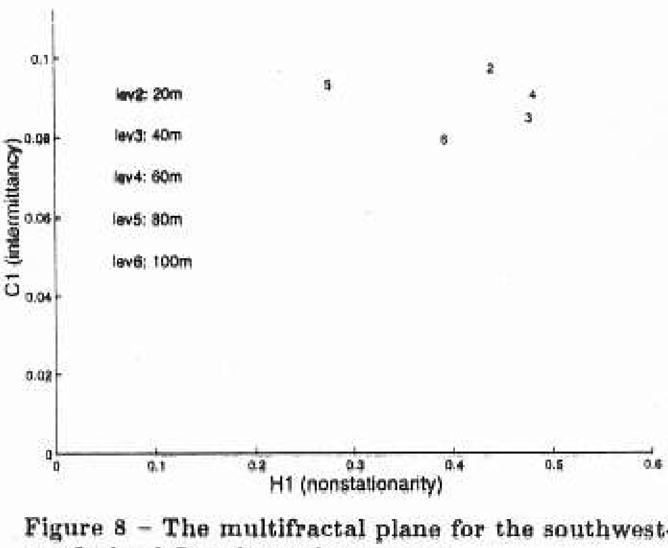

Figure 8 – The multifractal plane for the southwestern Iceland Sea thermal structure.